

### Question 3

$$\alpha = 0 \quad \beta = \gamma$$

$$RSS = \sum_{i=1}^n (y_i - \gamma x_i)^2$$

Derive OLS estimate for  $\gamma$

Take derivative with respect to  $\gamma$

$$\frac{dRSS}{d\gamma} = \sum_{i=1}^n 2(y_i - \gamma x_i)(-1) = 0$$

$$= \sum_{i=1}^n (y_i - \hat{\gamma} x_i)$$

$$= \sum y_i - n \hat{\beta}_0 - \hat{\beta}_1 \sum x_i = 0$$

$$0 = -\sum y_i + \hat{\gamma} \sum x_i$$

$$\hat{\gamma}_0 = \bar{y} - \hat{\gamma} \bar{x}$$

$\gamma_1$  equals:

$$\frac{d \text{RSS}}{d \hat{\gamma}_1} = \sum_{i=1}^n 2(Y_i - \hat{\gamma}_1 X_i) (-X_i) = 0$$

$$= \sum_{i=1}^n (Y_i - \hat{\gamma}_1 X_i) X_i = 0$$

$$= \sum_{i=1}^n Y_i X_i - \hat{\gamma}_1 \sum_{i=1}^n X_i^2 = 0$$

$$= \sum Y_i X_i - \hat{\gamma}_1 \sum X_i^2 = 0$$

$$= \frac{\sum Y_i X_i - \frac{1}{n} \sum Y_i \sum X_i}{\sum X_i^2 - \frac{1}{n} (\sum X_i)^2} = \hat{\gamma}_1$$

Yes it sum to zero